FAMOUS GREEK MATHEMATICS AND MATHEMATICIANS

(γεωμετρία = geometria)

Intended for Grades: 5 and 6

Subject: Science and Mathematics

Description: This project was designed to show students some accomplishments of famous Greek mathematicians. Among the mathematicians discussed are Pythagoras, Zeno, Archimedes, and Eratosthenes.

Mississippi Frameworks addressed:

Mathematics (5^{th})

- 1(c): Find the area of squares
- 2(c): Use appropriate tools to measure area, perimeter, and radius.
- 4(d): Model and distinguish between prime and composite numbers.

Mathematics (6^{th})

- 3(a): Calculate the area of parallelograms without using a calculator.
- 10(c): Distinguish between prime and composite number with and without using a calculator.

Science (5^{th})

- 9(b): Explore the effect of force on a object.

Materials: cut dowels, string, and fishing hooks.

Introduction:

We will study 4 famous Greek mathematicians: Pythagoras, Zeno, Archimedes, and Erathosthenes.. All of them made extraordinary achievements participating in the development of mathematics. We'll examine them as well as their achievements.

PYTHAGORAS

569-475 BC

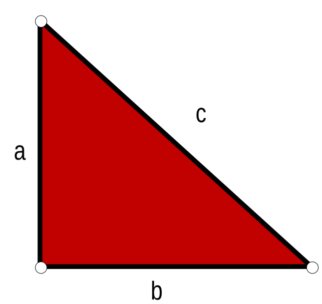
Born in Samos, Ionia

Pythagoras was a strange man. Not only was Pythagoras a mathematician, he was a mystic. He founded a community, which was both religious and scientific, built on mysticism. No one in the community ate beans, wore clothes made from animal skins, and everyone believed that this world is nothing more than patterns of whole numbers i.e. 1,2,3… and so on. Their creed was that everything in this world could be expressed as a ratio of whole numbers.

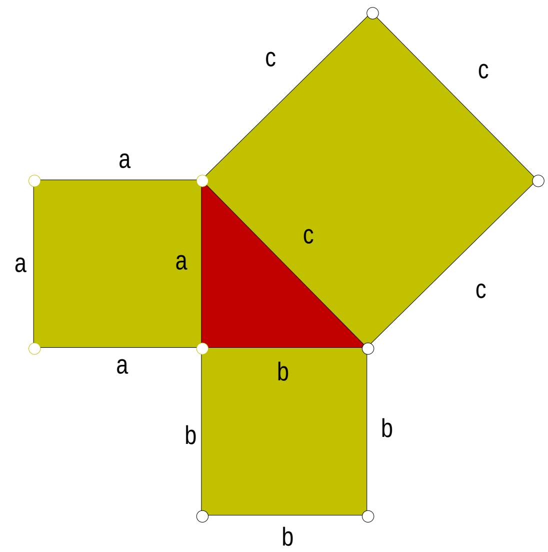
One of Pythagoras' most important achievements is his Pythagorean theorem which states that for a right triangle with sides of length a,b, and c where c is the length of the hypotenuse, $a^2+b^2=c^2$. However, this achievement was very disconcerting for him because it contradicted his creed. Indeed, take a=1 and b=1, then the hypotenuse is 🕑. Therefore the hypotenuse of this right triangle cannot be expressed with whole numbers.

Another one of Pythagoras' achievements is the idea of proof. Pythagoras started a movement where mathematicians would now have to prove their answers. This is where we begin. Let's examine why the Pythagorean theorem is true.

Consider a right triangle with side lengths of a,b, and c.



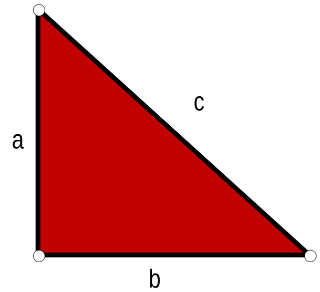
Now construct squares off of each side of the right triangle as in the following picture.



An important observation is this; when we use the Pythagorean theorem, $a^2+b^2=c^2$, what we're actually saying is that the area of the square with side lengths *a* added to the area of the square with side lengths *b* is equal to the area of the square with side lengths *c*. Test this out. You can do this one of two ways: you can draw this on paper using a ruler and then calculate the areas (or perhaps do this with the tiles on the floor) or you can use ***geometer sketchpad*** to draw the picture above and the program then will calculate the areas for you. Note that you may want to start with actual numbers for the side lengths instead of *a*,*b*, and *c*.

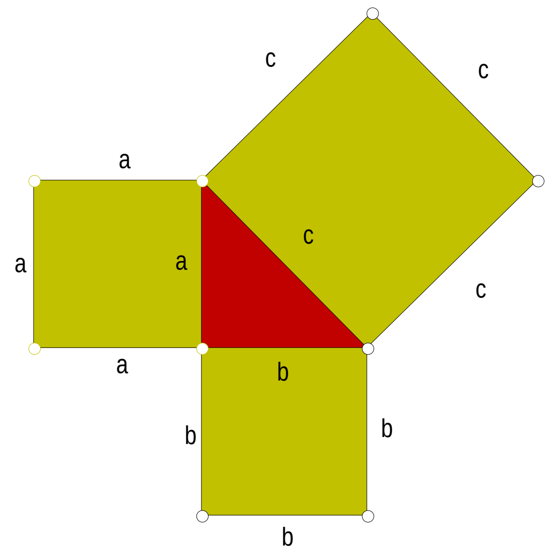
Understanding Pythagoras' Theorem using Geometer's Sketchpad

Given the following right triangle with sides of length a, b, and c where c is the hypotenuse:



we know from Pythagoras that $a^2+b^2=c^2$.

In the logic unit we looked at the following picture Euclid gave in his book called *The Elements*,

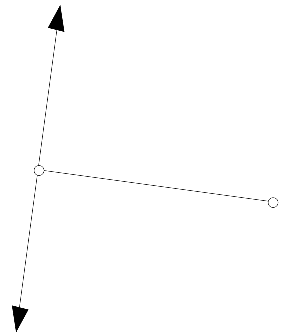


and from this picture we saw why Pythagoras's theorem is true.

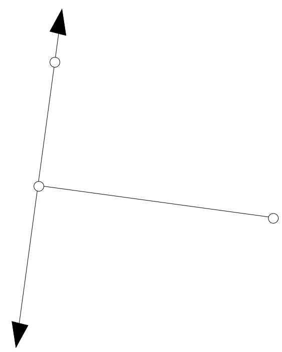
We can restate the Theorem in terms of area: the area of the two squares with sides a and b is equal to the area of the square with side c. We'll use Geometer's Sketchpad to create this picture and then calculate the areas to show that $a^2+b^2=c^2$.

To begin we first need to become acquainted with the tool bar which is located on the left-side of the screen. Go ahead and play around with it and become familiar with it. The first tool is the arrow and we'll use this to highlight different objects on the screen. In order to highlight more than one object at a time we'll need to hold down the shift key while we highlight. The second tool is the point tool. As you can see from the above picture, our squares and triangle all have points. The third tool is the circle tool and this will be important for us. We'll need to recall that the distance from the center of the circle to any point on the circle is the same distance. The fourth tool is the segment tool and finally the fifth tool will allow us to name our different objects. For example, our triangle above is labeled a, b, and c and this tool does this for us.

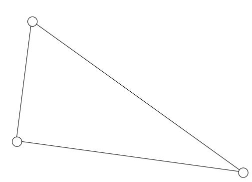
We first need to construct a right triangle. Therefore one of the angles needs to be 90 degrees. To create this all we need to do is draw two lines that are perpendicular to each other. With your segment tool draw a segment not too long. Notice that two points and a line connecting these points compose a segment. We can highlight any three of these objects. Highlight one point and the line. Remember that in order to highlight multiple objects we need to hold down the shift key while we do it. After you've highlighted, select the CONSTRUCT option at the top of your screen. This will give you the option of PERPINDICULAR LINES. Choose this option and a perpendicular line to the segment you drew should appear. Your picture should look like the following:



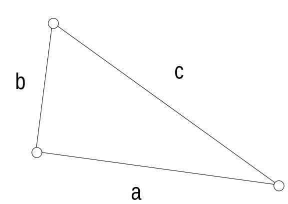
(Here the arrows just mean the line continues off this page.) Now you have two lines that are perpendicular to each other. Obviously we don't want our line to continue on like it does, therefore to correct this put a point on this line, anywhere, using your point tool.



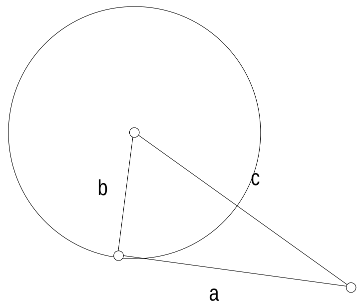
Now highlight only the line and choose DISPLAY at the top of the screen. From here you can choose HIDE LINE and what's left is your segment along with the point. Now all we must do is connect the points with the segment tool and we have a right triangle.



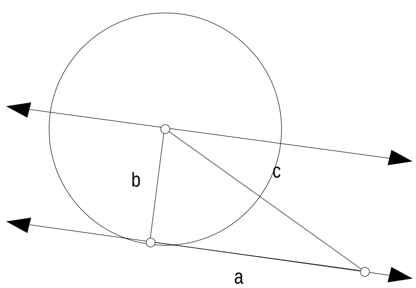
We can label the lines of this triangle a, b, and c with c being the hypotenuse.



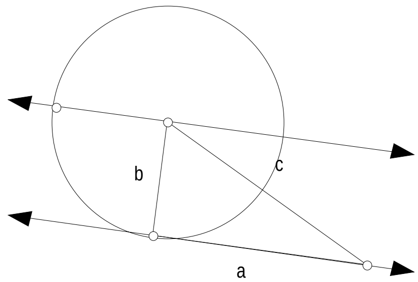
What we want to do next is extend squares off each side the triangle. This can be a little tricky because we need to make sure that the segments we construct for the squares are actually the same size and that we create 90 degree angles at the same time. To do this we'll use our circle tool. At the top-most point put a circle there and extend it down to the point below it, the point that shares the segments a and b. Your picture should look like the following;



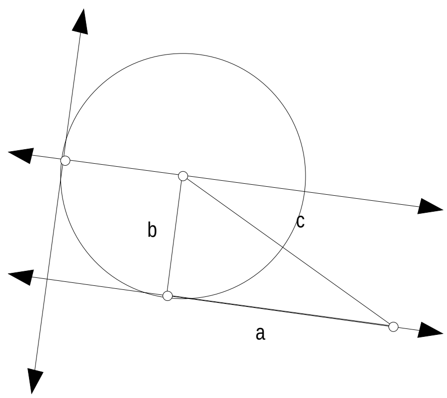
Therefore b represents the radius of this circle. So any other point on the circle will be the same length as b. We want to create segments coming off of the points of b to the circle but we need them to create a 90 degree angle with b. Therefore highlight b and both its points and choose CONSTRUCT and then choose PERPENDICULAR LINES again.



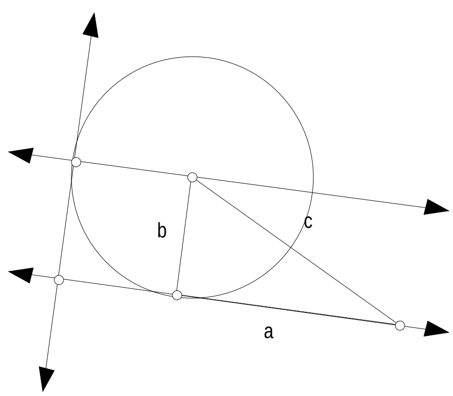
Now put a point where the perpendicular line and circle meet.



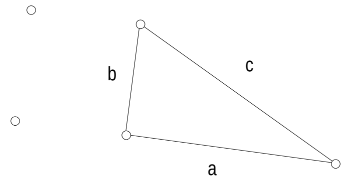
As you can see we almost have our square extended off of b. We need one last line, so highlight the top line along with the point you just made and choose again PERPENDICULAR LINE.



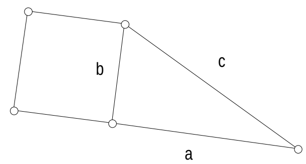
This line will meet the bottom line, put a point there.



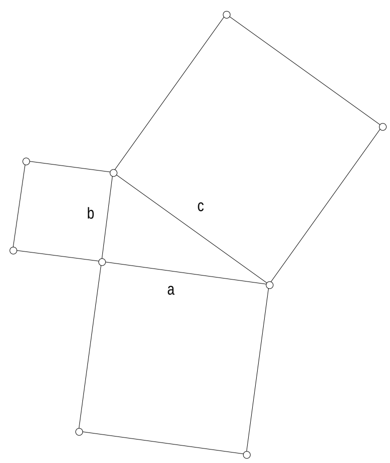
Again, we don’t want our lines continuing along like they do, we want segments and we don’t want the circle. So highlight the lines and circles (make sure you don’t highlight any of the triangle) and choose DISPLAY and HIDE OBJECTS like before.



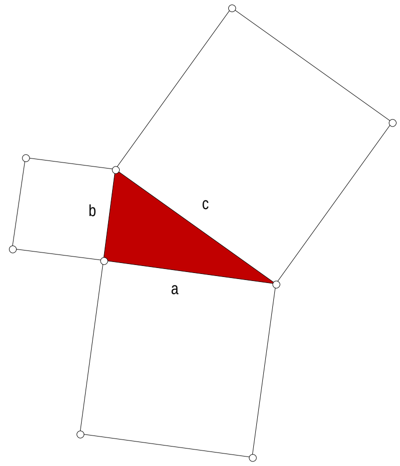
Now connect the dots.



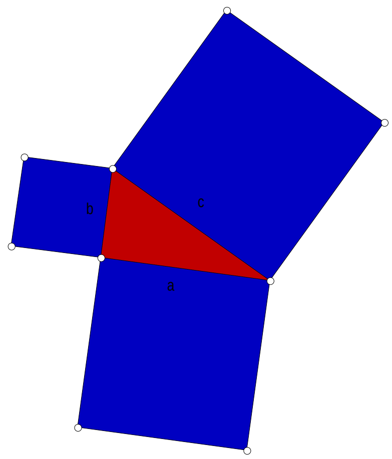
We'll do the same construction to create squares that extend off of a and c. And so we get the following picture;



Next we'll calculate the area of each. To do this we must create a polygon interior. Let's do the triangle first. Highlight the three points of the triangle. When you've done this choose CONSTRUCT and then choose POLYGON INTERIOR. The interior of your triangle should be shaded. Now with your arrow tool you highlight and un-highlight the interior of your triangle. Go ahead and highlight the interior of your triangle. Choose DISPLAY and then COLOR. Here you can pick a different color for your interior.



Do the same thing for the squares.



Now we want to calculate the area of each square. Highlight the interior of one of your squares. Choose MEASURE at the top of your screen and then choose AREA. This will calculate the area for you and put it in the top left hand side of your screen. It will look something like this;

Area AGHB = 1.52 inches ^{2}

Do the same thing for the rest of your squares.

Area DEAC = 0.42 inches ^{2}

Area CKLB = 1.96 inches ^{2}

As you can see, 1.54+.42=1.96.

ZENO

490BC – 425BC

Born in Elea, Luciana

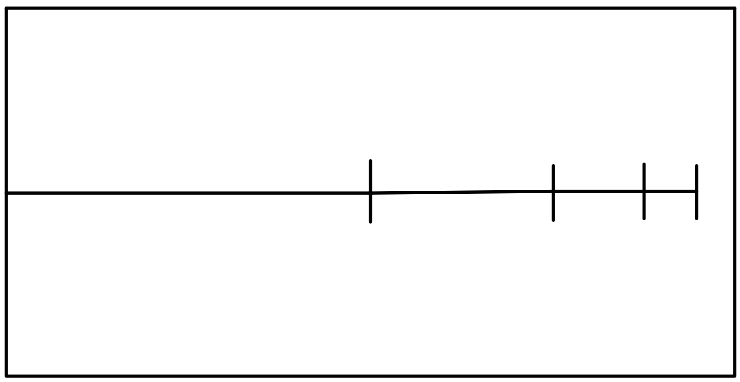
Zeno of Elea was more of a philosopher than a mathematician, however, he did challenge the mathematics of his time. Zeno is well known for his paradoxes. These paradoxes support his philosophy that "all is one". Everything we see and touch is actually one thing in and of itself. Therefore he believed that change is impossible. For example, when we walk we change our position. Zeno believed that this was an illusion, that actually we were traveling nowhere. He used the following paradox to support this.

Consider Achilles and a tortoise and suppose that they were in a race. Now Achilles is known in Greek history to be the fastest of all men. So naturally Achilles would win this race even if the tortoise had a head start. However, Zeno gave an interesting representation of the race.

Let the Tortoise have a 10-meter head start. Now let the race begin. Achilles will catch the tortoise but first Achilles will reduce the 10 meters between him and the tortoise to 5 meters. Then Achilles will reduce the 5 meters to 2.5 meters and then to 1.25 meters and then to .625 meters and so on. We keep halving. We can do this indefinitely so that Achilles never actually catches the tortoise and since the tortoise is moving forward the tortoise will actually win the race. Now we know that in reality Achilles would win, so what's wrong with this representation of the race? This challenged the mathematics of

Zeno's time because no mathematician could explain why this representation is wrong. This is why Zeno believed that change or motion is impossible.

Here's another way to see this and you can do this in your classroom. Begin walking from one side of the classroom to the other. However walk in such a way that you stop in the middle between you and the other side. So walk according to the diagram where the small vertical lines mean you've stopped.



Will you ever reach the other side of the classroom? This is actually how walk, first we reach one half way point then we reach another half way point and another and so on. What's going on??

ARCHIMEDES

287 BC – 211BC

Born in Syracuse, Sicily

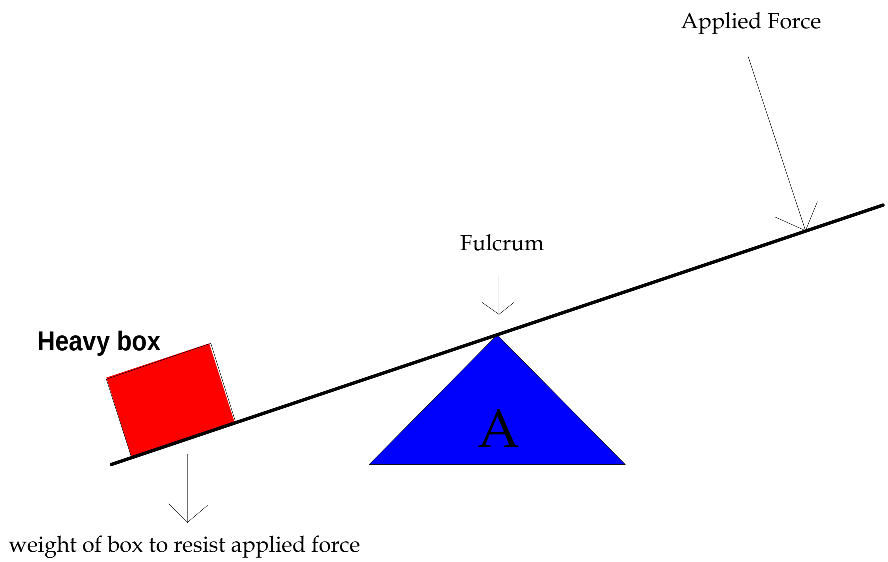
Archimedes is sometimes referred to as one of the smartest men that ever lived. He is considered to be one of the top three mathematicians, which is an astounding accomplishment since there have been many brilliant mathematicians since 287 BC. We'll examine one of his accomplishments, war machines, though there are many.

Archimedes' war machine: The Claw

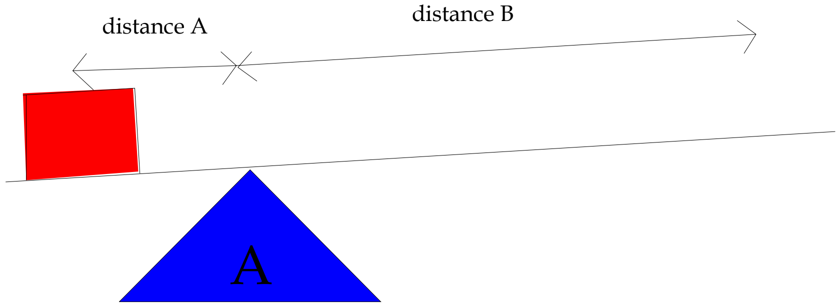
The claw consisted of a grappling hook that lied just beneath the water, a lever, and perhaps some counterweights that sat on top of a wall by the sea. As the enemy ship approached the wall, the ship would float over the grappling hook, and then the lever with the counterweight was pulled down raising the ship into the air. When the men let go of the lever the ship would crash down into the rocks breaking into pieces and hurling the enemy out into the sea. This machine proved to be very successful when Rome was invading Greece.

Archimedes had proclaimed that, given another planet to stand on, he could move the Earth with a lever. The lever with counterweights (or pulley) allowed men to lift very heavy objects including ships. Archimedes knew this and so was able to engineer the most powerful weapon during his time.

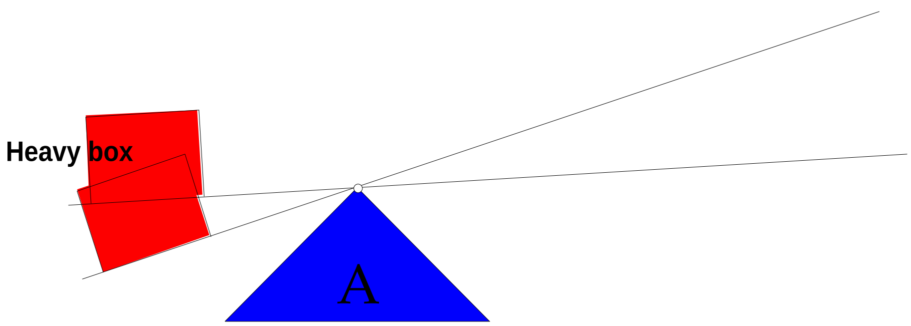
The lever (a rigid bar) consists of three parts, the fulcrum, applied force, and resistance force. The fulcrum is the point about which the bar pivots or swings. We supply the applied force and the object we're moving supplies the resistance force. For example, say we wanted to lift a heavy box with a long piece of wood rested on some other object, we'll call it object A. The long piece of wood is our rigid bar and the point at which the wood rotates or swings is the place where the wood rests on object A and so is our fulcrum. At one end of the wood is where we'll apply force and so is the applied force. At the other end is where the heavy box rests and resists the force we apply and so is the resistance force.



With this scenario we could potentially lift the heavy box but there's a way in which we could lift this heavy box easily. In fact, if we set this up just right, it may only take one person to lift the box. Consider the distance from the heavy box to the fulcrum (we'll call this distance A) and the distance from the fulcrum to the other end (likewise distance B). The greater distance B is than distance A, the easier it will be to lift the heavy box.



Note that the heavy box moves a much shorter distance than the other end. This means that the force we apply to the wood is multiplied and so will lift the heavy box.



Note the distance the heavy box moves compared to the distance moved at the other end.

Your task is to build a miniature claw using the materials given to you. First do some research on the Claw using the Internet. The site listed below is a good place to start. Then explain the lever used in your miniature claw, i.e. explain where the rigid bar, fulcrum, applied force, and resisted force is and explain how counterweights help with the applied force. Finally of course is the fun part, test to see if they work!

1. www.math.nyu.edu/~crorres/Archimedes/Claw/harris/rorres_harris.pdf, 2001

ERATOSTHENES

276 BC – 194 BC

Born in Cyrene, North Africa

Eratosthenes is perhaps best known as an all around scholar. He was a mathematician, in fact a quite well recognized mathematician accredited with an accurate measurement of the Earth's circumference and a sieve for prime numbers. He was also a philosopher and poet. However, the problem with being an all around scholar is that in each subject he fell short of the highest rank. His nickname was beta (being second to alpha) and pentathlos meaning an all around athlete who places second in the events but never first.

The sieve of Eratosthenes is a prime number filter. We'll take numbers 1 through *n*, where *n* can be any number you like, and put them in the sieve. The sieve will keep only prime numbers and let all other numbers fall out. You can think of it as a spaghetti strainer. After boiling the noodles, they and the water is dumped into a strainer to separate the two and as we know, the hot water passes through the strainer. The prime number sieve is like a spaghetti strainer in that the prime numbers are the noodles and all other numbers is the hot water.

Consider numbers 1 through 25. First we examine 1. By definition, 1 is not prime number since it does not have enough divisors, i.e. 1 only divides itself and nothing else. So in the following table 1 is red since it is not prime.

1	2	3	4	5
6	7	8	9	10
11	12	13	14	15
16	17	18	19	20
21	22	23	24	25

Now we examine 2. The only number dividing 2 is itself and 1 and so is prime. Now the sieve removes all multiples of 2, i.e. 4, 6, 8 and so on will now be red.

1	2	3	4	5
6	7	8	9	10
11	12	13	14	15
16	17	18	19	20
21	22	23	24	25

As you can see, the sieve now has eliminated many numbers from being prime. The next number after 2 not red is 3 and so must be prime. We now remove all multiples of 3 by making them red.

1	2	3	4	5
6	7	8	9	10
11	12	13	14	15
16	17	18	19	20
21	22	23	24	25

The next number not red is 5 and so must be prime and thus remove all multiples of 5. We continue in this fashion for all numbers that remain black. The following table then gives all the prime numbers between 1 and 25 where the red numbers are composite.

1	2	3	4	5
6	7	8	9	10
11	12	13	14	15
16	17	18	19	20
21	22	23	24	25

Of course there's not so many steps to take since we have only 25 numbers. Use the sieve of Eratosthenes to find all prime numbers between 1 and 100. Notice that we use consecutive numbers starting with 1 when using this sieve. Can we use this sieve for consecutive numbers starting with another number other 1 or 2? Can we use this for numbers that are not consecutive?

There are many other famous mathematicians the students could study and the mathematicians presented here have many more accomplishments. Hence there's much more to research and study if the teacher wishes to do so.

References:

1. O'Connor, J J and E F Robertson. *Eratosthenes of Cyrene*. www-history.mcs.st-andrews.ac.uk/Mathematicians/Eratosthenes.html. January 1999.
2. Rorres, Chris and Harris, Harry. *A Formidable War Machine: Construction and Operation of Archimedes' Iron Hand.*, www.math.nyu.edu/~crorres/Archimedes/Claw/harris/rorres_harris.pdf, 2001.
3. Zeno of
4. Archimedes of
5. Pythagoras of

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